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# A novel computational technique for the geometric progression of powers of two

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**Abstract:** This paper presents a new mathematical theorem on the geometric progression whose each term designates a power of two.

**Keywords**: computation, power of two, geometric progression.

## 1. Introduction

Sequences and series of powers of two play a key role in research areas such as computer science, information systems, electrical & electronics, medicine, computational biology, etc. In this study, a new mathematical theorem is introduced using a geometric progression of powers of two.

## 2. Novel computational technique

**Theorem:** $\sum_{i=k}^{n-1} 2^i = 2^n - 2^k$ where $k \in \mathbf{Z}$, set of integers.

**Proof 1**

$2^n = 2^n$

$2^n = 2^{n-1} + 2^{n-1}$

$2^n = 2^{n-1} + 2^{n-2} + 2^{n-2}$

Similarly, we can continue this mathematical expression as follows

$2^n = 2^{n-1} + 2^{n-2} + 2^{n-3} + \cdots + 2^i + \cdots + 2^k + 2^k$

$\therefore \sum_{i=k}^{n-1} 2^i = 2^n - 2^k$

where $k \in \mathbf{Z}$, set of integers.

**Proof 2**

We can also prove the theorem $\sum_{i=k}^{q-1} 2^i = 2^q - 2^k$ by induction.

**Basis**

Let $k = 0$.

As per the theorem, the result is $\sum_{i=0}^{q-1} 2^i = 2^q - 2^0 = 2^q - 1$.

Now, the same result using actual geometric series formula can be computed as follows:

$$\sum_{i=0}^{q-1} 2^i = \frac{2^q - 1}{2 - 1} = 2^q - 1$$

**Inductive hypothesis**

Let us assume that the theorem is true for $k = \pm(n\text{-}1)$.

**Inductive step**

We must prove that the inductive hypothesis is true for $k = \pm n$

As per the theorem, the results are shown as

$$(i)\ \sum_{i=n}^{q-1} 2^i = 2^q - 2^n \quad \text{for k} = n$$

$$(ii)\ \sum_{i=-n}^{q-1} 2^i = 2^q - 2^{-n} \quad \text{for } k = -n$$

Now, the same results using actual geometric series formula are computed as follows:

$$(i)\ \sum_{i=n}^{q-1} 2^i = \sum_{i=0}^{q-1} 2^i - \sum_{i=0}^{n-1} 2^i = \frac{2^q - 1}{2-1} - \frac{2^n - 1}{2-1} = 2^q - 2^n$$

$$(ii)\ \sum_{i=-n}^{q-1} 2^i = \sum_{i=-n}^{-1} 2^i + \sum_{i=0}^{q-1} 2^i = \sum_{i=1}^{n} \frac{1}{2^i} + \sum_{i=0}^{q-1} 2^i = \frac{\frac{1}{2} - \frac{1}{2^{n+1}}}{1 - \frac{1}{2}} + \frac{2^q - 1}{2-1}$$

$$i.e.\ \sum_{i=-n}^{q-1} 2^i = 2^q - 2^{-n}$$

Thus, the inductive hypothesis is true for $k = \pm n$.

Hence, the theorem is proved**.**

**4. Conclusion**

In the research study, an innovative computational theorem has been formulated along with detailed proof. This new theorem will be very useful for researchers involving in broad research areas such as computer science, information systems, electrical & electronics, biomedicine, computational biology, etc.